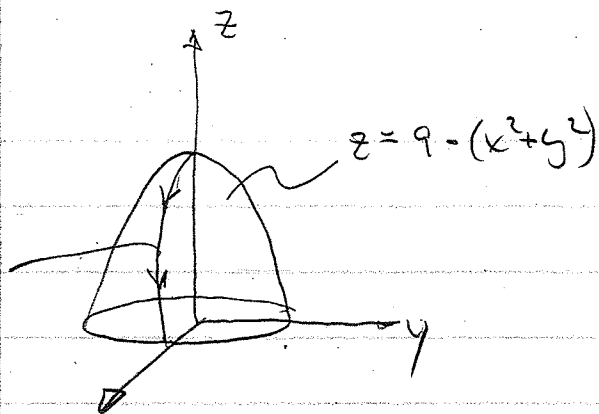


3

path



a) $\underline{v}(t) = \underset{\substack{\uparrow \\ x(t)}}{t} \hat{i} + \underset{\substack{\uparrow \\ y(t)}}{0} \hat{j} + \underset{\substack{\uparrow \\ z(t)}}{f(t)} \hat{k}$ where $x(t), y(t), z(t)$ must satisfy

$$z = 9 - x^2 - y^2$$

$$\therefore z(t) = 9 - t^2 - 0^2 = f(t)$$

so $\underline{v}(t) = t \hat{i} + 0 \hat{j} + (9 - t^2) \hat{k}$ \leftarrow

$$\underline{v}(t) = \hat{i} + 0 \hat{j} + (-2t) \hat{k}$$

b) Time when $z(t) = 9 - t^2 = 0 \Rightarrow t = 3 \leftarrow$

c) $S = \int_{t=0}^3 |\underline{v}| dt = \int_{t=0}^3 \sqrt{1 + 4t^2} dt$ $u = 2t \quad du = 2dt$

$$= \frac{1}{2} \int_{u=0}^6 \sqrt{1 + u^2} du$$

so

$$2S = \left[\frac{u}{2} \sqrt{1 + u^2} + \frac{1}{2} \ln(u + \sqrt{1 + u^2}) \right]_{u=0}^6$$

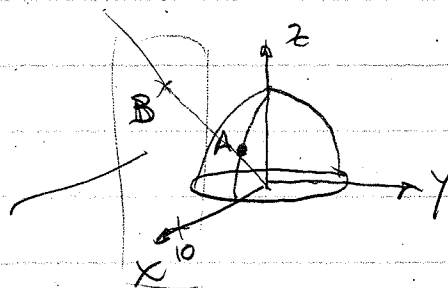
or

$$S = \frac{1}{4} [6\sqrt{37} + \ln(6 + \sqrt{37})] \leftarrow$$

d) at time $t = 2$, pt A on path is

$$\underline{v}(2) = 2 \hat{i} + 0 \hat{j} + 5 \hat{k}$$

plane
 $x = 10$



③ cont.

So line through origin + A is

$$L(s) = 2s\hat{i} + 0\hat{j} + 5s\hat{k}$$

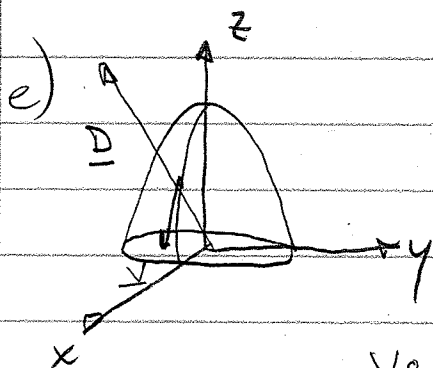
which has direction $\underline{D} = 2\hat{i} + 0\hat{j} + 5\hat{k}$

To find s when L intersects plane $x=10 = 2s$

$$\text{so } s = 5$$

Thus coord. of B are $L(5) = 10\hat{i} + 0\hat{j} + 25\hat{k}$

or $(10, 0, 25) \leftarrow$



Want \angle between $\underline{V}(t=2) = \hat{i} - 4\hat{k}$
and $\underline{D} = 2\hat{i} + 0\hat{j} + 5\hat{k}$

$$\underline{V} \cdot \underline{D} = -18 = |\underline{V}| |\underline{D}| \cos \theta$$

$$\cos \theta = \frac{-18}{|\underline{V}| |\underline{D}|} = \frac{-18}{\sqrt{17} \sqrt{29}}$$

$$\theta = \cos^{-1} \left(\frac{-18}{\sqrt{17} \sqrt{29}} \right) \leftarrow$$

