③ $I = \iint_R (3x^2 + 14xy + 8y^2)\,dx\,dy$ using $u = 3x + 2y$ $v = x + 4y$

which inverts to

a) $x = \frac{2}{5}u - \frac{1}{5}v$ $y = \frac{-1}{10}u + \frac{3}{10}v$

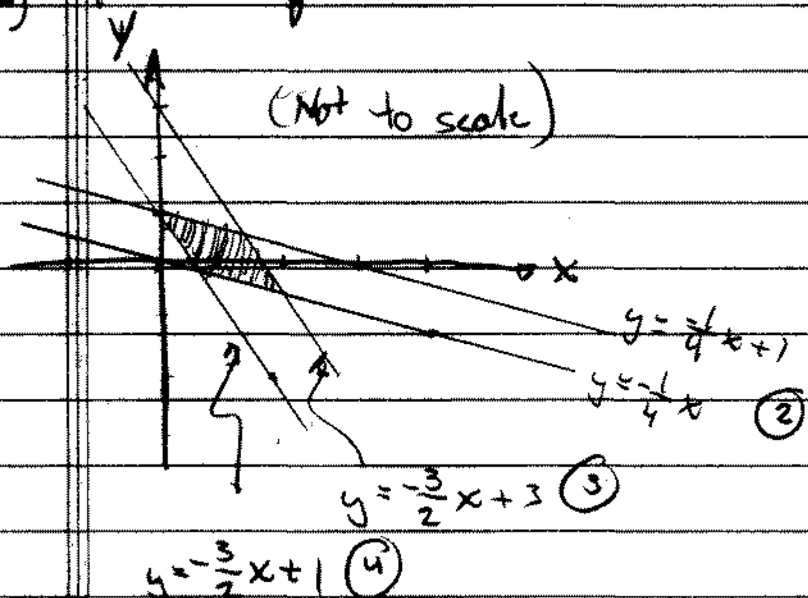
and has Jacobian

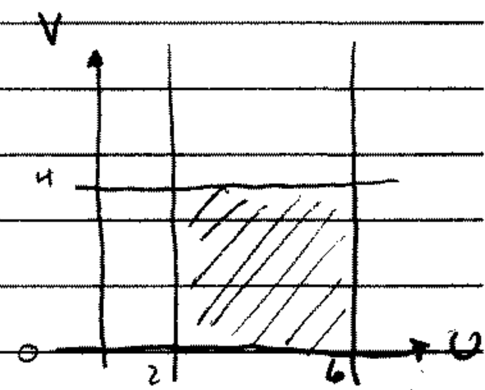
$$J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 2/5 & -1/5 \\ -1/10 & 3/10 \end{vmatrix}$$

$$= \frac{5}{50} = 1/10$$

Also note that $3x^2 + 14xy + 8y^2 = uv$

b) 1^{st} & 4^{th} quadrant version is





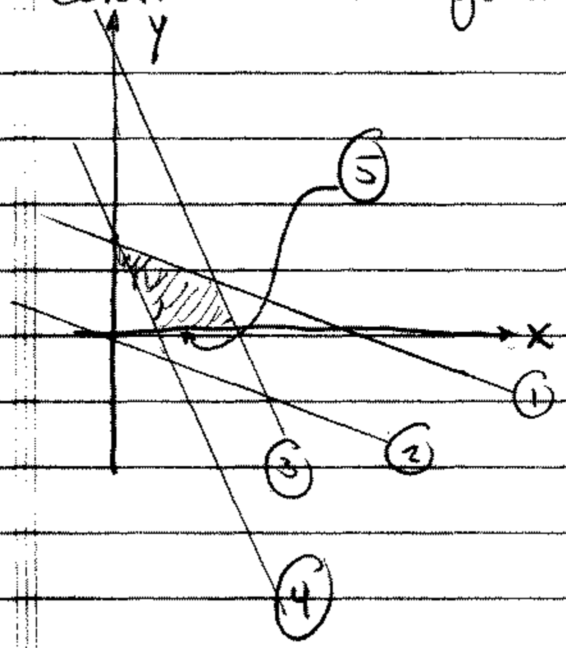
Boundary ①	②	③	④
$4y = -x + 4$	$4y = -x$	$2y = -3x + 6$	$2y = -3x + 2$
$4y + x = 4$	$4y + x = 0$	$2y + 3x = 6$	$2y + 3x = 2$
$v = 4$	$v = 0$	$u = 6$	$u = 2$

c) $$I = \int_{v=0}^{4} \int_{u=2}^{6} (uv) \left|\frac{1}{10}\right| du\,dv = \frac{1}{10}\int_{v=0}^{4} \frac{v}{2}(32)\,dv$$

d) $$= \frac{1}{10} 16 \frac{1}{2} 16 = 64/5$$

3 cont' 1^{st} quad. version

b)



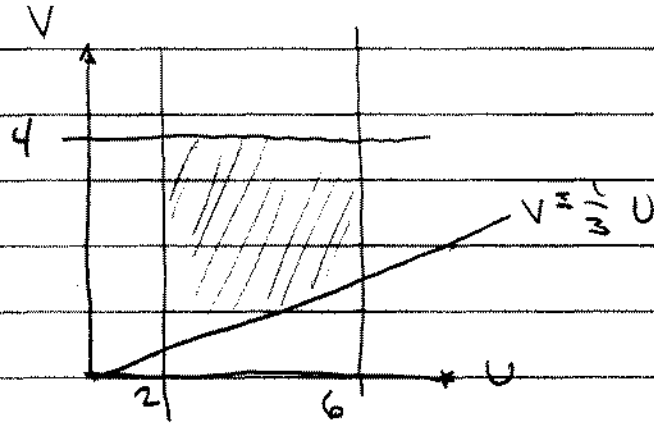
Boundary 5 is

$y = 0$

$\frac{-1}{10}u + \frac{3}{10}v = 0$

$+u = 3v$

b)



c) $$I = \int_{u=2}^{6} \int_{v=u/3}^{4} uv \left|\frac{1}{10}\right| dv\,du = \frac{1}{10}\int_{u=2}^{6} \frac{u}{2}\left(16 - \frac{u^2}{9}\right) du$$

$$= \frac{1}{2\cdot 10}\left(8u^2 - \frac{u^4}{4\cdot 9}\right)\Bigg|_2^6 = \frac{1}{20}\left(8\cdot 32 - \frac{1280}{4\cdot 9}\right)$$

d) $$= 992/90 \leftarrow$$